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WRITING MATHEMATICS WELL

Communicating mathematics well is an important part of doing mathematics. As you write up your homework solutions, keep these things in mind:

- **Write in sentences.**
  Complete thoughts are sentences that end in periods. You may still highlight important equations by displaying them, but even displayed equations should have punctuation! Use paragraphs to separate important ideas.
- **Use helpful connective phrases.**
  "If", "then", "so", "therefore", "we see that", "recall that", ...
- **Your audience is other students in the class who have not seen this problem before.**
  Remind the reader of any relevant facts from class or the book. Your solution should give adequate detail so that the reader can follow your solution.
- **It is possible to write too much!**
  If you write out every triviality, the reader may get lost in the details. This is not good writing, either. (In particular, really trivial calculations need not be shown.)
- **Avoid shorthand.**
  Don't use arrows, and write out 'for all', 'there exists'.
- **You may wish to outline your problem-solving strategy at the beginning of the problem.**

Example. Here are two different solutions to the same problem. Which one is easier to understand?

- **Solution 1**

$$(0-3)^2 + (x-2)^2 = 25$$
$$3^2 = 9 + (x^2 - 4x + 4) = 25$$
$$x^2 - 4x - 12$$
$$(x-6)(x+2) \Rightarrow x = -2, 6 \quad x > 0 \quad x = 6$$

- **WHY THIS IS POORLY WRITTEN:**
  - You don't know what problem the writer was solving.
  - You can't tell what's an assumption and what's a conclusion.
  - Where does one thought end and another begin? There are no sentences!
  - In the 2nd line: combining two thoughts can create untruths (3^2 is 9 but it isn't 25).
  - The 3rd line dangles; what's being asserted here? It's not a sentence.
  - What's the relationship between all these phrases? Connective phrases would help!

- **Solution 2**

Problem:

Find a point in the plane on the positive x-axis that has a distance 5 from the point (2, 3).

Solution

The desired point is (6. 0).
To find this, we note if $(x, 0)$ is a solution, then x must satisfy the equation
$(x-2)^2 + (0-3)^2 = 25$, which follows from the planar distance formula between the points (x, 0) and (2, 3). It follows that $x^2 - 4x + 13 = 25$. Then

$$x^2 - 4x - 12 = 0$$

Factoring, we obtain $(x-6)(x+2) = 0$,
Satisfied by either $x = -2$ or $x = 6$. Since we assumed x > 0 and y = 0, we see (6, 0) is the desired point.

- **WHY THIS IS WELL-WRITTEN:**
  - The writer described the problem, and strategy for solution.
  - Every thought is a complete sentence with subject and verb (the "equals" sign is a verb).
  - She answered the question right at the beginning. (Boxing answers is customary.)
  - Notice even the equations have punctuation (comma, periods) as they are part of sentences.
  - She highlighted important ingredients, displayed important equations, avoided trivial algebra.

Writing well will benefit you, too! It helps you structure your own thinking, and you will thank yourself when you re-read your solutions later.