LESS TRANSPARENT

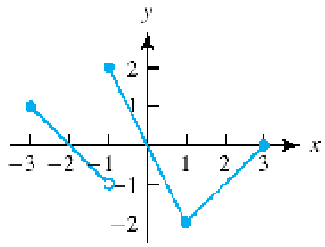
EXAMPLE L: TIMED CALCULUS EXAMINATION

Chapter 4

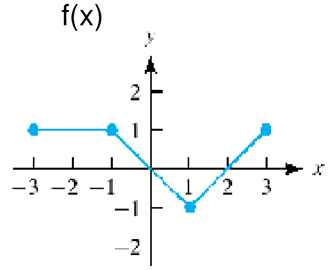
Name: ________

Make sure you show your work and that solutions are neat and easy to follow! Use exact answers!

1. Use the figures below to find the limit: lim $x \to -2$ $\frac{g(x) - 1}{f(x)}$



Graph of f



Graph of g

2. Find each of the limits:

a) lim $x \longrightarrow \infty$ $\frac{1-x^2}{2x^2+5}$

b) lim $x \longrightarrow \infty$ $x^2 e \; -\frac{x^2}{2}$

c) lim $x \longrightarrow \infty$ $\frac{x^3}{3 - \sin x}$

d) lim $x \longrightarrow 0$ $\frac{1-\cos(5x)}{4x + 3x^2}$

EXAMPLE L continued: TIMED CALCULUS EXAMINATION

3. Find all critical points and label all maximums, minimums, or neither for $f(x) = x + \sin x$ on the interval $0 \le x \le 2\pi$.

4. The mass of a cube in grams is $M = x^3 + 0.1x^4$, where x is the length of one side in centimeters. If the length is increasing at a rate of 0.02 cm/hr, at what rate is the mass of the cube increasing when its length is 5 cm?

5. Find values of a and b so that the function $y = axe^{-bx}$ has a local maximum at the point (2, 10).

6. Set up the following problem (DO NOT SOLVE!!) In other words, find the equation that you would take the derivative of - in terms of one variable. An ordinary soft drink can has a volume of 355 cubic centimeters. Find the dimensions that would minimize the cost of making the can.

7. Set up the following problem (DO NOT SOLVE!!) In other words, find the equation that you would take the derivative of - in terms of one variable. Which point on the line $y = -\cos x$ is closest to the (1,1)?

8. You are driving towards the 1063-foot Eiffel tower at a rate of 60 feet per second. How fast is the angle of elevation changing when you are 1000 feet away?

 Trina Palmer

MORE TRANSPARENT

Revised EXAMPLE L: UNTIMED CALCULUS PORTFOLIO

Due: Friday, November 1 Used by permission of Trina Palmer

Purposes: The purposes of this assignment include:

- improve your mathematical writing
- demonstrate your mathematical thinking
- communicate your mathematical reasoning
- demonstrate your proficiency with applications of the derivative.

Derivatives are an instantaneous rate of change. Derivatives model any quantity that changes with respect to another quantity such as velocity or carbon emission rates.

Tasks:

For this Derivative Application Assignment, you are to write up 5 problems with clarity and detail.

- You must pick one problem from section 4.7.
- You must pick one problem from section 4.8.
- All 5 questions must come from different sections.
- You must rewrite the question before your work.
- You must include a cover page with the problems you chose. See the sample provided.

Here are the problems you can choose from:

- Section 4.2: 54, 56, 58
- Section 4.3: 42
- Section 4.4: 46, 64
- Section 4.5: 60, 64
- Section 4.6: 48
- Section 4.7: 18, 19, 22, 28
- Section 4.8: 32, 34

Criteria:

Each of the 5 problems is worth 20 points. The rubric for each problem is below. 100 total points are available for this project, so this portfolio is equivalent to a test score.

	Proficient	Emerging	Needs Improvement
Calculus Accuracy	Includes most steps and steps are accurate (9)	missing a few steps and/or some steps are inaccurate (6)	many missing steps and/or many inaccurate steps (1)
Algebraic Accuracy	Includes most steps and steps are accurate (5)	missing a few steps and/or some steps are inaccurate (3)	many missing steps and/or many inaccurate steps (1)
Mathematics Language	language/terminology is correct and mostly correct math language (5)	reasoning is mostly correct and mostly correct mathematics language (3)	many incorrect reasoning (1)
Problem Restatement	rewrites problem correctly (1)	rewrites problem incorrectly (0.5)	does not include problem statement (0)

SAMPLE PRODUCT: (Optimization)

Problem: Let $b > 0$. Find where $f(x) = xe^{-bx}$ has a maximum(s).

Answer: $\boxed{\boldsymbol{f(x) = xe^{-bx}} \textbf{ has a maximum at } \boldsymbol{x = \frac{1}{b}}}$

Solution: To find the maximum(s) of a function, we want to first find the critical points of the function. Critical points are where the derivative is zero or does not exist. In our problem, $f(x)$ is smooth so the derivative exists everywhere. The derivative of $f(x)$ is

$$f'(x) = x(-be^{-bx}) + e^{-bx}.$$

We get this by using the product rule for derivatives where the two functions being multiplied are x and e^{-bx}. The derivative of x is 1 and the derivative of e^{-bx} is $-be^{-bx}$ by using the chain rule. Thus the critical points of $f(x)$ are the x-values when $f'(x) = x(-be^{-bx}) + e^{-bx}$ equals 0. Setting $^{t}(x) = 0$, we get

$$f'(x) = x(-be^{-bx}) + e^{-bx} = 0.$$

Factoring the e^{-bx} out of both terms we get

$$e^{-bx}(-bx + 1) = 0.$$

Thus, either $e^{-bx} = 0$ or $(-bx + 1) = 0$. Since e^{-bx} is always positive, there is no x-value that makes $e^{-bx} = 0$. Solving $-bx + 1 = 0$ for x, gives us $x = \frac{1}{b}$. At this point, we just know that $x = \frac{1}{b}$ is a critical point of $f(x)$. Now, we need to show whether this is a maximum, minimum or neither. To do this we could use either the first derivative test or the second derivative test. Because there is an unknown constant, b, we will use the second derivative test. Using the product and chain rules again, we get that the second derivative is

$$f''(x) = x(b^2e^{-bx}) - be^{-bx} - be^{-bx}.$$

Plugging the critical point, x = 1 into the second derivative gives

$$f''(\tfrac{1}{b}) = \tfrac{1}{b}(b^2e^{-b^{1/b}}) - be^{-b^{1/b}} + e^{-b^{1/b}} = be^{-1} - be^{-1} - be^{-1} = -be^{-1}$$

$f''(\frac{1}{b}) = be^{-1}$ is a negative number because b > 0 and $e^{-1} > 0$. Since $f''(\frac{1}{b})$ is a negative number this means that f(x) is concave down at x = $(\frac{1}{b})$ which by the second derivative test, there is a maximum at x = $(\frac{1}{b})$.

 Trina Palmer

SAMPLE COVER PAGE

Your name
MAT 1110 – CALCULUS I

Portfolio for Applications of the Derivative

Problems included are:

1. Section 4.2 #58
2. Section 4.4 #64
3. Section 4.5 #50
4. Section 4.7 #22
5. Section 4.8 #32